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Recent Advances in Rate-Time Analysis – Application to Production Data from Unconventional Oil Reservoirs

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12/10/2015

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Outline

- Problem Description
- Objective
- Method
- Results and Discussion
- Summary
- Conclusion

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Problem Description

- Hydraulically fractured wells in unconventional reservoirs exhibit
  - Long periods of transient flow
  - Production data is expected to exhibit multiple time scales
  - Uncertain well stimulation outcomes (fracture dimensions, orientation, geometry)
- Effect of 'secondary' physics on producing characteristics
  - Desorption
  - Stress dependent permeability and porosity
  - Combination of secondary physics

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Problem Description

	Decline type	Rate Function	Cumulative Function
Arps (1945)	Hyperbolic 0<b<1	$q_i(1+bD_it)^{-\frac{1}{b}}$	$\frac{q_i}{D_i(b-1)}\left[(1+bD_it)^{1-\frac{1}{b}}-1\right]$
$D=-\frac{1}{q}\frac{dq}{dt}$ $\quad b=-\frac{d}{dt}\left(\frac{1}{D}\right)$			
b = 0, Exponential decline 0 < b <1, Hyperbolic decline b = 1, Harmonic decline			
Valko (2009)	SEDM	$\hat{q}_i e^{-\left(\frac{t}{\tau}\right)^n}$	$\frac{\hat{q}_i\tau}{n}\left[\Gamma\left(\frac{1}{n}\right)-\Gamma\left(\frac{1}{n},\left(\frac{t}{\tau}\right)^n\right)\right]$
Ilk et al. (2010)	Power law	$\hat{q}_i e^{(-D_\infty t-\hat{D}_i t^n)}$	
Clark et al. (2011)	Logistic growth model	$\frac{kt^{n-1}}{a+t^n}$	$\frac{knat^{n-1}}{(a+t^n)^2}$

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Objective

Develop "SIMPLE" rate – time models for unconventional oil reservoirs

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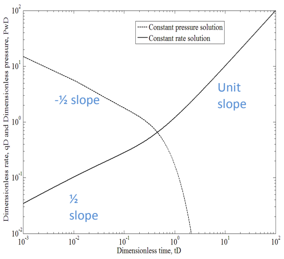
Correlation Functions for Empirical Model Parameters and the Reservoir and Well Properties

Method

- Integrated data analysis to identify
  - Producing characteristics (based on linear flow theory)
  - Predominant flow regime(s)
- Model based analysis to estimate model parameters using
  - Logistic growth model (LGM)
  - Parallel flow model (PFM)
- Investigate existence of any correlation between model parameters and reservoir/well completions properties
- Use design of experiments, numerical reservoir simulation and multivariate regression analysis to develop correlations

Method

Theoretical Basis for Data Analysis



- Constant pressure solution

$$q_D = -2\sum_{n=1}^{\infty}(-1)^n e^{-\left[\left[\frac{(2n-1)\pi}{2}\right]^2 t_D\right]} sin\frac{(2n-1)\pi}{2}$$

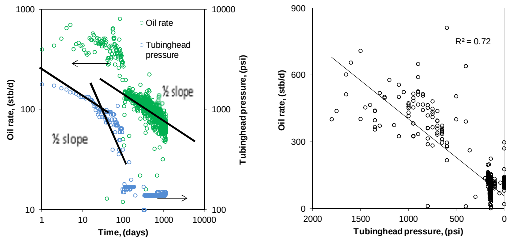
- Constant rate solution

$$p_{wD} = -\frac{t_D}{2} - \frac{1}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}(1-e^{-n^2\pi^2 t_D})\cos[n\pi]$$

- On log-log plot
  - Half slope ≡ TLF
  - Exponential ≡ BDF
  - Unit slope ≡ BDF

NOTE
TLF: Transient linear flow
BDF: Boundary dominated flow

Results and Discussion



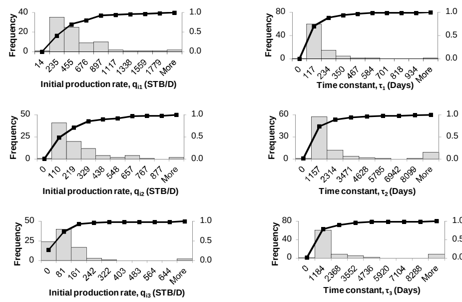
- Observed slopes ranging from one-half, one, three and a half
- Predominant slope observed was one-half

Method: Model Based Analysis

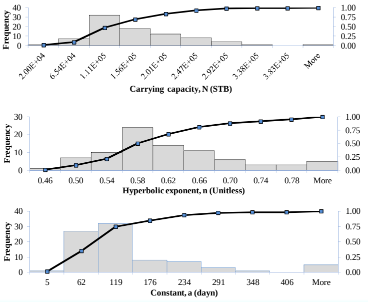
	LGM	PFM
Rate, q	$\frac{N a n t^{n-1}}{(a+t^n)^2}$	$\sum_{k=1}^{N_e} q_{i_k} e^{-\frac{t}{\tau_k}}$
Cumulative, N_p	$\frac{N t^n}{a+t^n}$	$\sum_{k=1}^{N_e} f_k \tau_k (1-e^{-\frac{t}{\tau_k}})$
Fitting function	$\min e = \sum_{m=1}^{M} \left[q_m^{data} - q_m^{\text{model}} \right]^2 + \sum_{m=1}^{M} \left[N_{p,m}^{data} - N_{p,m}^{\text{model}} \right]^2$	$\min e = \sum_{m=1}^{M} \left[f_m - \left[\sum_{k=1}^{N_e} f_k e^{-\frac{t}{\tau_k}} \right]_m \right]^2 + \sum_{m=1}^{M} \left[N_{p,m}^{data} - \left[\sum_{k=1}^{N_e} f_k \tau_k (1-e^{-\frac{t}{\tau_k}}) \right]_m \right]^2$ subject to: $\sum_{k=1}^{N_e} f_k = 1$

LGM: Logistic Growth Model, PFM: Parallel Flow Model

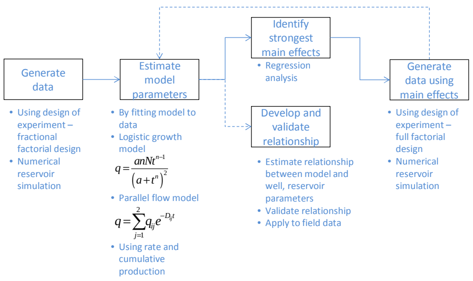
Results and Discussion: PFM



Results and Discussion: LGM



Method: Correlation Development



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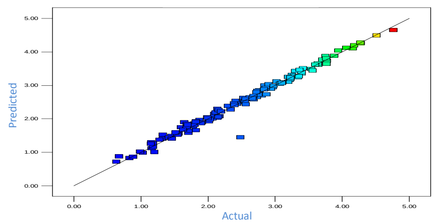
Results and Discussion

- Initial Production Rate 1, qi1



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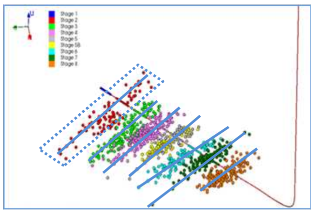
An Approximate Analytical Solution to the Double – porosity Model and its Application to Unconventional Oil Reservoirs

Method

Processed micro-seismic data

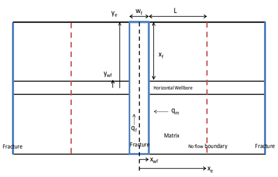


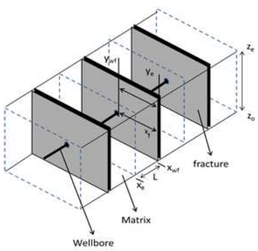
Source: esg solutions

Micro-seismic event: Shear/slip events that occur as result of hydraulic fracture

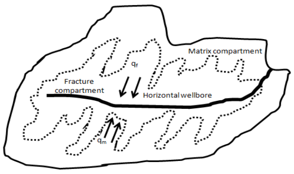
Method

- Idealized Conceptual Model





Method



New problem

$$\left(v_{p_m} c_t\right)_m \frac{d\bar{p}_m}{dt} = -q_m$$

$$\left(v_{p_f} c_t\right)_f \frac{d\bar{p}_f}{dt} = -q_f + q_m$$

Initial condition:

$$q_f(t=0) = q_{fi}$$

$$q_m(t=0) = 0$$

Advantage(s)

- Independent of fracture geometry/orientation and shape of drainage volume
- Much easier to solve

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Method

Solution

- Using linear flow theory
- Valid across all flow regimes

Relationship between rate and average pressure

Series decomposition of ODE

- Block diagonal system
- Eigen-value decomposition

- Sum independent solutions
- Closed form approximate solution

Simplification

$$q_f = \left[\frac{q_{fi}\gamma}{(\gamma-\rho)} e^{\lambda_2 t} + \frac{q_{fi}\gamma\sqrt{\pi}}{4\sqrt{-\lambda_2 t}\,(\gamma-\rho)} erfc(3\sqrt{-\lambda_2 t})\right] - \left[\frac{q_{fi}\rho}{(\gamma-\rho)} e^{\lambda_1 t} + \frac{q_{fi}\rho\sqrt{\pi}}{4\sqrt{-\lambda_1 t}\,(\gamma-\rho)} erfc(3\sqrt{-\lambda_1 t})\right]$$

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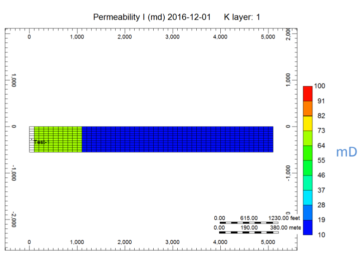
Results and Discussion

Model Validation with Numerical Simulation



Parameter	value
Δx x Δy x Δz	100 x 50 x 50
Grid	51 x 11 x 1
Depth, D (ft)	2000
Thickness, h (ft)	50
Compartment 1	
permeability, k_f (md)	70
Porosity	0.3
volume, (ft^3)	8250000
Compartment 2	
permeability, k_m (md)	10
Porosity	0.3
volume, (ft^3)	33000000

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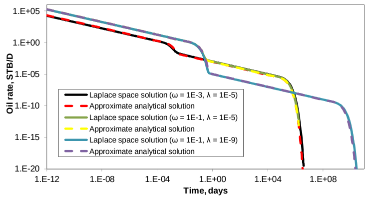
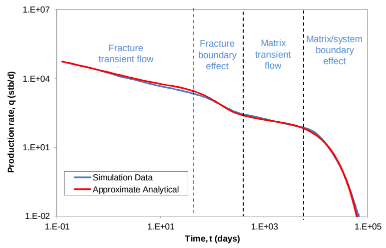
Results and Discussion

- Model Validation with Laplace Space Solution



Results and Discussion

Model Validation with Numerical Simulation

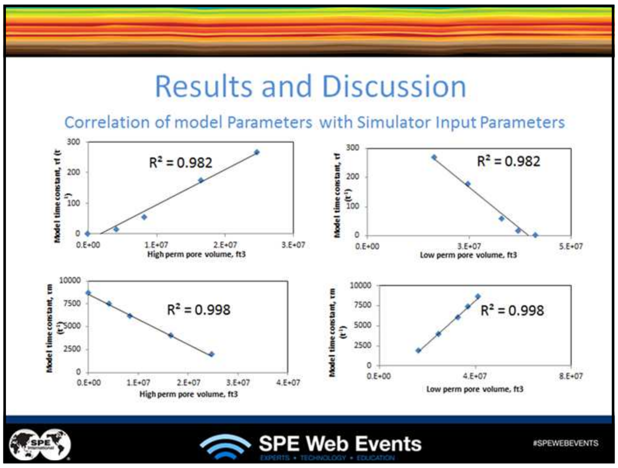


Parameter	Value
q_{fi} (b/d)	4695.61
τ_f (day)	56.00
τ_m (day)	6087.00
T_x/l_f	0.03670
λ_1 (day^{-1})	-0.00016
λ_2 (day^{-1})	-0.01852
Υ (day^{-1})	-27.0051
ρ (day^{-1})	1.0090
$\Upsilon/(\Upsilon-\rho)$	0.9640
$\rho/(\Upsilon-\rho)$	-0.0360

Results and Discussion

Analysis of Model Parameters

Case	CMG V_{pf} (ft^3)	CMG V_{pm} (ft^3)	τ_f (days)	τ_m (days)	$1/\lambda_1$ (days)	$1/\lambda_2$ (days)	q_{fi} (b/d)
1	8.25×10^6	3.30×10^7	56	6087	6312.4	54	4695.61
2	1.65×10^7	2.48×10^7	176	3998	4247.01	165.68	2695.61
3	2.48×10^7	1.65×10^7	267	1900	2462.16	206.04	2080.02
4	4.13×10^6	3.71×10^7	15	7455	7639.51	14.64	8195.61
5	0.00×10^0	4.13×10^7	-	8634	8855.91	-	7095.61


Results and Discussion
Correlation of model Parameters with Simulator Input Parameters
R² = 0.982
High perm pore volume, ft3
R² = 0.982
Low perm pore volume, ft3
R² = 0.998
High perm pore volume, ft3
R² = 0.998
Low perm pore volume, ft3
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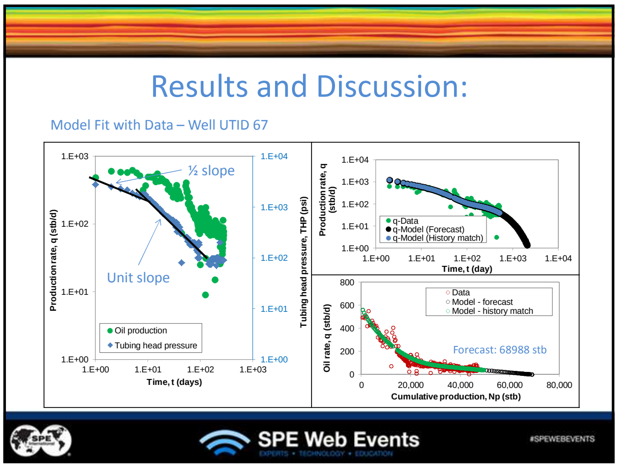
Results and Discussion:
Model Fit with Data – Well UTID 67
½ slope
Unit slope
Oil production
Tubing head pressure
Production rate, q (stb/d)
Tubing head pressure, THP (psi)
Time, t (days)
q-Data
q-Model (Forecast)
q-Model (History match)
Time, t (day)
Data
Model - forecast
Model - history match
Forecast: 68988 stb
Oil rate, q (stb/d)
Cumulative production, Np (stb)
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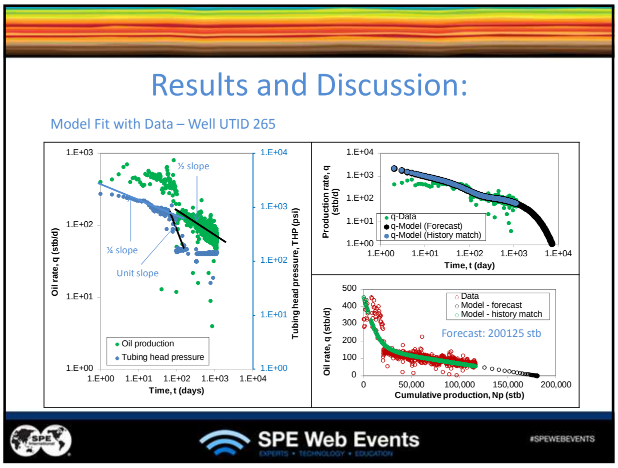
Results and Discussion:
Model Fit with Data – Well UTID 265
½ slope
¼ slope
Unit slope
Oil production
Tubing head pressure
Oil rate, q (stb/d)
Tubing head pressure, THP (psi)
Time, t (days)
q-Data
q-Model (Forecast)
q-Model (History match)
Time, t (day)
Data
Model - forecast
Model - history match
Forecast: 200125 stb
Cumulative production, Np (stb)
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Results and Discussion

Fit parameters	UT-ID265	UT-ID67
τ_f (days)	1,480.16	470.13
τ_m (days)	53.37	27.26
t_f (days)	~90	~10

- Fracture and matrix volume larger in UT-ID265
- Larger Reserves from UT-ID265

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Summary

- Multiple time scales observed in production data
- Developed correlations between model parameters and reservoir/well properties
- Developed a "simple" rate time model
  - valid for all flow regimes (transient and pseudo steady state)
  - explains the time scale observed
- Eliminates the limitations of empirically derived models

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Conclusion

- All the models fit production data "well"
- The approximate double porosity model performs better
  - Model parameters are functions of reservoir/well properties
  - Indicative of relative drainage volume
- Useful information contained in early-time production data

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Method: Governing Equations

Fracture Equations

$$\frac{\partial^2 p_f}{\partial z^2}+\frac{\partial^2 p_f}{\partial y^2}+\frac{\partial^2 p_f}{\partial x^2}=\frac{(\phi\mu c_t)_f}{k_f}\frac{\partial p_f}{\partial t}$$

$$p_f(x,y,0)=p_i$$

$$p_f(x,y_{wf},t)=p_{wf}$$

$$\left.\frac{\partial p_f}{\partial x}\right|_{x=0}=\left.\frac{\partial p_f}{\partial x}\right|_{y=y_e}=\left.\frac{\partial p_f}{\partial x}\right|_{z=z_0}=\left.\frac{\partial p_f}{\partial x}\right|_{z=z_e}=0$$

$$\left.\frac{k_f}{\mu}\frac{\partial p_f}{\partial x}\right|_{x_{wf}}=\left.\frac{k_m}{\mu}\frac{\partial p_m}{\partial x}\right|_{x_{wf}}$$

Matrix Equations

$$\frac{\partial^2 p_m}{\partial z^2}+\frac{\partial^2 p_m}{\partial y^2}+\frac{\partial^2 p_m}{\partial x^2}=\frac{(\phi\mu c_t)_m}{k_m}\frac{\partial p_m}{\partial t}$$

$$p_m(x,y,0)=p_i$$

$$\left.\frac{\partial p_m}{\partial x}\right|_{x=x_e}=\left.\frac{\partial p_m}{\partial x}\right|_{y=y_e}=\left.\frac{\partial p_m}{\partial x}\right|_{y=y_{wf}}=\left.\frac{\partial p_m}{\partial x}\right|_{z=z_e}=\left.\frac{\partial p_m}{\partial x}\right|_{z=z_0}=0$$

$$\left.\frac{k_f}{\mu}\frac{\partial p_f}{\partial x}\right|_{x_{wf}}=\left.\frac{k_m}{\mu}\frac{\partial p_m}{\partial x}\right|_{x_{wf}}$$

Results and Discussion: PFM

Mean	398	182	62	381	2.7×10^4	2.1×10^4
Median	289	119	20	57	448	222
Standard Deviation	391	201	116	2570	1.6×10^5	1.1×10^5
Kurtosis	5	5	21	86	45	39
Skewness	2	2	4	9	7	6
Range	2×10^3	1×10^3	7×10^2	2×10^4	1×10^6	7×10^5
Minimum	14	0	0	0	0	0
Maximum	2×10^3	1×10^3	7×10^2	2×10^4	1×10^6	7×10^5

Parameters appear to be log normally distributed

τ_2 and τ_3 appear to come from the same distribution (Two time scales)

Results and Discussion: PFM

Mean	133135.35	0.62	5897.30
Median	111940.75	0.58	75.96
Standard Deviation	69011.60	0.23	52876.12
Kurtosis	3.12	63.29	84.00
Skewness	1.32	7.47	9.16
Range	408248.15	2.13	484738.60
Minimum	20000.00	0.46	4.52
Maximum	428248.15	2.59	484743.12

Parameter are log normally distributed

Hyperbolic constant varies between 0 and 1

Results and Discussion

LGM Parameter Correlations	
$N^{0.17}$	$5.74+2.63\times10^{-2}h+2.1\times10^{-4}L_w+1.1\times10^{-3}x_f$
n	$6.6\times10^{-1}-8.2\times10^{-4}k_f+2.03\times10^{-5}k_m+8.8\times10^{-6}L_w$
$a^{-0.26}$	$1.5\times10^{-1}+2.9\times10^{-4}k_f-5.5\times10^{-6}k_m$

PFM Parameter Correlations	
$\ln(q_{i1})$	$6.4\times10^{-2}+6.7\times10^{-3}k_f+4.0k_m+1.8\times10^{-4}p_i-5.4\times10^{-4}p_{wf}-2.1\times10^{-3}h-3.9\phi-1.1\times10^{-4}L_w$ $+7.4\times10^{-5}k_fh+3.6\times10^{-7}k_fL_w+2.3\times10^{-2}k_mL_w+4.7\times10^{-8}p_ip_{wf}+5.4\times10^{-6}hL_w-2.1\times10^{-8}k_fhL_w$
$\tau_1^{1.12}$	$6.9\times10^{2}-3.8k_f+4.2\times10^{5}k_m-16.1h+1.3\times10^{4}\phi-1.5\times10^{-2}L_w-5.9\times10^{6}k_m\phi-4.5hL_w$
$\ln(q_{i2})$	$-1.7+2.0\times10^{-3}k_f+1.7\times10k_m+2.4\times10^{-4}p_i-2.1\times10^{-4}p_{wf}+1.6\times10^{-2}h+6.4\phi+6.1\times10^{-5}L_w+5.9\times10^{-1}k_fk_m$
$\tau_2^{0.67}$	$7.6\times10^{2}-9.7\times10^{-1}k_f-7.4\times10^{3}k_m+2.3h+4.3\times10^{-3}L_w-2.3\times10^{2}k_fk_m+18.4k_mL_w-6.5\times10^{-4}hL_w$

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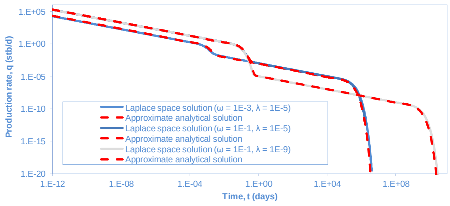
Comparison to Laplace Space Solution



- Model reproduces Laplace space solution
- Same producing characteristics (macroscopic equations)

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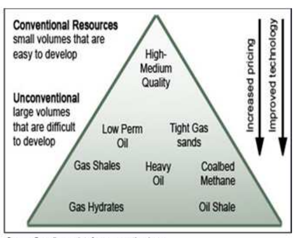
Problem Description

- Huge deposits of unconventional resources



Source: Canadian society for unconventional gas

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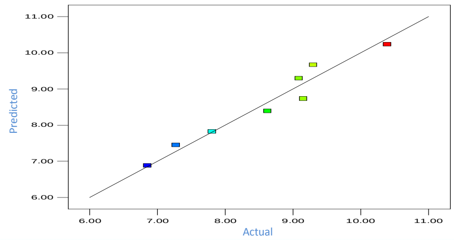
Results and Discussion

- Carrying capacity, N

$$N^{0.17} = 5.74 + 2.63\times10^{-2} h + 2.1\times10^{-4} L_w + 1.1\times10^{-3} x_f$$



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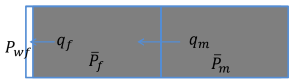
Model Derivation

- Re- formulated Problem as a systems of ODEs:

$$(V_p C_t)_f \frac{d\bar{p}_f}{dt} = -q_f + q_m$$

$$(V_p C_t)_m \frac{d\bar{p}_m}{dt} = -q_m$$



$$\bar{p}_f = p_{wf} + \frac{4}{\pi^2}(p_i - p_{wf}) \sum_{n=1}^{\infty} \frac{2}{(2n-1)^2} e^{-\frac{(2n-1)^2 \pi^2 k_f t}{4\phi\mu c_t x_f^2}}$$

Initial condition:

$$@\, t = 0, \bar{p}_{1_n} = \bar{p}_{2_n} = p_i \Rightarrow q_{2i_n} = 0$$

$$\bar{p}_m = \bar{p}_f + \frac{4}{\pi^2}(p_i - p_{wf}) \sum_{n=1}^{\infty} \frac{2}{(2n-1)^2} e^{-\frac{(2n-1)^2 \pi^2 k_m t}{4\phi\mu c_t L^2}}$$

$$\sum_{n=1}^{\infty} \begin{bmatrix} \dot{q}_f \\ \dot{q}_m \end{bmatrix} = \sum_{n=1}^{\infty} \begin{bmatrix} \dot{q}_{f_n} \\ \dot{q}_{m_n} \end{bmatrix} = \sum_{n=1}^{\infty} (2n-1)^2 \begin{bmatrix} -\frac{1}{\tau_f} & \frac{1}{\tau_f} \\ \frac{T_x}{j_f \tau_f} & -\left(\frac{T_x}{j_f \tau_f} + \frac{1}{\tau_m}\right) \end{bmatrix} \begin{bmatrix} q_{f_n} \\ q_{m_n} \end{bmatrix}$$

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Results and Discussion: PFM

	q_{i1} (STB/D)	q_{i2} (STB/D)	q_{i3} (STB/D)	τ_1 (Days)	τ_2 (Days)	τ_3 (Days)
Number of stages	0.47	0.43	0.31	-0.07	-0.10	-0.06
Lateral length (ft)	-0.19	-0.21	-0.30	-0.13	0.15	0.03
Spacing (acres)	0.32	0.35	0.22	-0.34	0.10	0.09
Initial water saturation (fraction)	-0.35	-0.54	-0.30	0.18	-0.18	-0.02
Porosity (fraction)	0.28	0.21	0.27	-0.09	-0.12	0.22
TVT (ft)	-0.40	-0.59	-0.40	0.29	-0.03	-0.11
Net to gross (fraction)	0.15	0.21	0.29	-0.14	-0.18	0.32
Overpressure (psi)	0.18	0.33	0.19	-0.02	0.07	-0.12
Pressure (psi)	0.18	0.33	0.19	-0.02	0.07	-0.12
Depth (ft)	0.15	0.30	0.17	-0.05	0.14	-0.14
Average injection pressure (psig)	0.35	0.36	0.16	-0.21	0.09	0.02
Total fluid injected	0.46	0.44	0.30	-0.37	0.08	0.12
Sand (lbs)	0.48	0.40	0.36	0.02	-0.12	-0.06

- Linear relationship highlighted $|\rho| > 0.2$
- Some level of non linear correlation observed from cross plots

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Results and Discussion: LGM

	N (STB)	n	a
Number of stages	0.14	0.02	-0.02
Lateral length (ft)	0.00	-0.13	-0.09
Spacing (acres)	0.25	0.02	0.04
Initial water Saturation (fraction)	-0.11	-0.09	-0.01
Porosity (fraction)	0.46	-0.12	-0.05
TVT (ft)	-0.05	-0.09	0.02
Net to gross (fraction)	0.19	0.02	0.01
Overpressure (psi)	-0.04	0.10	0.00
Pressure (psi)	-0.04	0.10	0.00
Depth (ft)	-0.03	0.10	0.02
Average injection pressure (psig)	0.23	-0.02	0.09
Total fluid injected	0.39	-0.02	-0.02
Sand injected (lbs)	0.26	0.02	0.01

- Linear relationship highlighted $|\rho| > 0.2$
- Little/no correlation observed from cross plots

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Results and Discussion

Strongest Main Effect Variables

Property	Parallel flow model				Logistic growth model		
	q_{i1} (STB/D)	q_{i2} (STB/D)	τ_1 (Day)	τ_2 (Day)	N (STB)	n	a
Fracture half length, x_f (ft)					•		
Fracture permeability, k_f (md)	•	•		•		•	
Initial oil saturation, S_{oi} (fraction)							
Initial reservoir pressure, P_i (psi)	•	•		•			
Flow well pressure, P_{wf} (psi)		•				•	
Matrix permeability, k_m (md)	•	•				•	•
Number of fracture cluster per stage							
Number of fracture stages							
Porosity, ϕ (fraction)		•					
Thickness, h (ft)	•	•			•		
Well length, L (ft)	•	•	•	•	•	•	•
Width, w (ft)							
Fracture spacing, (ft)							
Viscosity, μ (cp)							
Compressibility, c_t (psi^{-1})							

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