



High-Resolution Subsurface Imaging Under Geologic Uncertainty with Emerging Signal Processing Techniques

Behnam Jafarpour, USC



SPE Web Events

EXPERTS • TECHNOLOGY • EDUCATION

JOIN THE CONVERSATION

 @SPE_EVENTS

 #SPEWEBEVENTS



June 26, 2013





Related R&D Grand Challenge

Research Objective:

Exploiting emerging sparse signal processing techniques for high resolution reservoir imaging

Related R&D Grand Challenge:

Transparent Earth: High Resolution Subsurface Imaging

2

High-Resolution Subsurface Imaging with Emerging Signal Processing Techniques

Behnam Jafarpour, USC



SPE Web Events

EXPERTS • TECHNOLOGY • EDUCATION

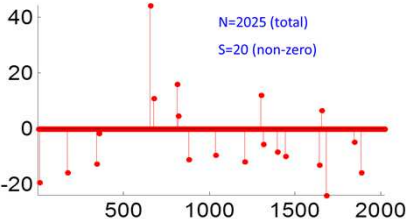
#SPEWEBEVENTS



Sparsity

$$\|v\|_0 = \#\{i, v_i \neq 0\}$$
$$\|v\|_0 \ll \dim(v)$$

Sparsity



N=2025 (total)

S=20 (non-zero)

3

High-Resolution Subsurface Imaging with Emerging Signal Processing Techniques

Behnam Jafarpour, USC



SPE Web Events

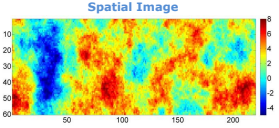
EXPERTS • TECHNOLOGY • EDUCATION

#SPEWEBEVENTS

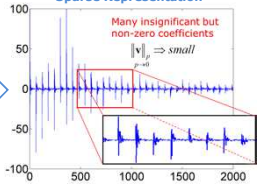
Sparsity and Subsurface Images

Sparsity of spatially distributed reservoir properties

Spatial Image



Sparse Representation



4

High-Resolution Subsurface Imaging with Emerging Signal Processing Techniques

Behnam Jafarpour, USC



#SPEWEBEVENTS

Overview

- Reservoir History Matching (Inverse Modeling)
- Parameterization of Reservoir Properties
- Sparsity-Promoting History Matching
- Sparse Geologic Dictionaries
- Geologic Uncertainty
- Field Application

5

High-Resolution Subsurface Imaging with Emerging Signal Processing Techniques

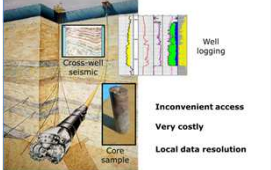
Behnam Jafarpour, USC

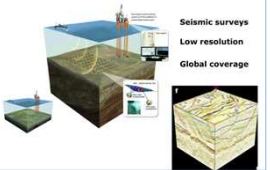


#SPEWEBEVENTS

Predictive Model Construction

Descriptive static model(s) → Predict dynamic behavior/response → Integrate dynamic data (calibration)





Seismic surveys
Low resolution
Global coverage

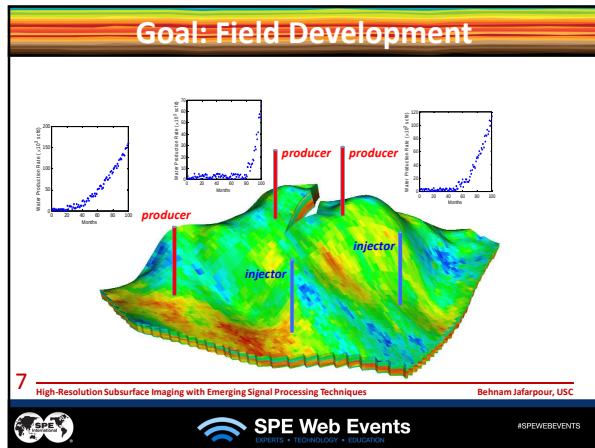
6

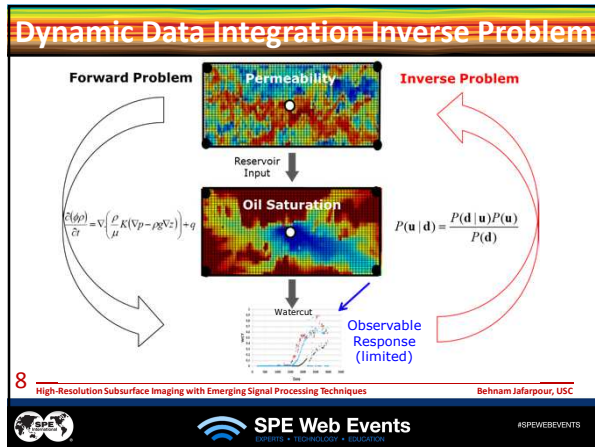
High-Resolution Subsurface Imaging with Emerging Signal Processing Techniques

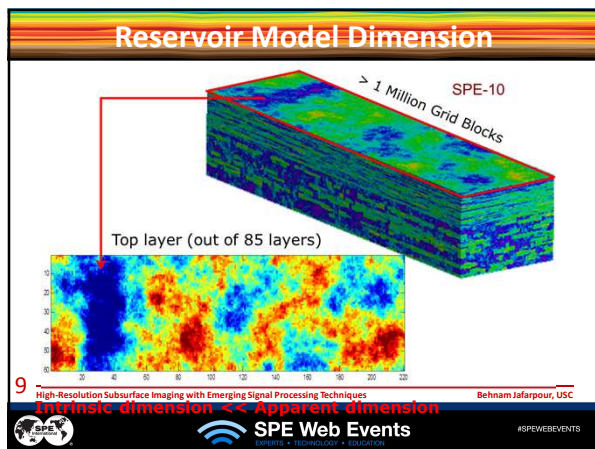
Behnam Jafarpour, USC



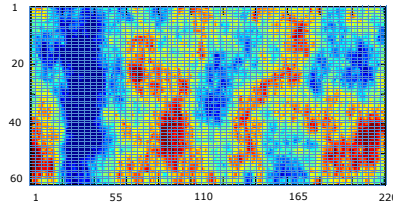
#SPEWEBEVENTS







Need for Reduced Reservoir Descriptions



$N=220 \times 60 = 13,200$ cells

Grid-based description issues:

- Over-parameterization → Non-unique solutions
- Dimensionality → computational stability & complexity
- Preserving geologic continuity

10 High-Resolution Subsurface Imaging with Emerging Signal Processing Techniques Behnam Jafarpour, USC

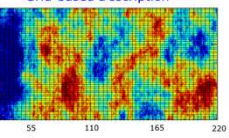
  **SPE Web Events**
EXPLORATION • TECHNOLOGY • EDUCATION #SPEWEBEVENTS

Focusing on Reservoir Connectivity

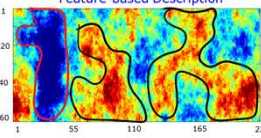
Number of grid cells
 $N=220 \times 60 = 13,200$ cells

Focus on important features (connectivity)

Grid-based Description



Feature-based Description



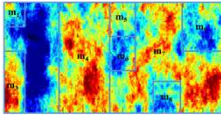
11 High-Resolution Subsurface Imaging with Emerging Signal Processing Techniques Behnam Jafarpour, USC

  **SPE Web Events**
EXPLORATION • TECHNOLOGY • EDUCATION #SPEWEBEVENTS

Spatial Parameterization (e.g., Zonation)

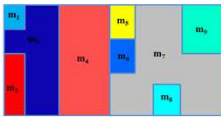
Zonation

Parameter size reduction through regionalized description
(Since 1965, Jacquard & Jain, Jahns,...)



Main Issues:

- Identifying regions/zones
- Sharp boundaries → geologically unrealistic



12 High-Resolution Subsurface Imaging with Emerging Signal Processing Techniques Behnam Jafarpour, USC

  **SPE Web Events**
EXPLORATION • TECHNOLOGY • EDUCATION #SPEWEBEVENTS

Transform-Domain Parameterization

Compact representations in compressive bases

Full-Rank Representation

$$\mathbf{u} = \sum_{i=1}^N \phi_i v_i$$

N expansion terms

Low-Rank Approximation

$$\hat{\mathbf{u}} \approx \sum_{s=1}^S \phi_s v_s$$

$S (<N)$ expansion terms

13 High-Resolution Subsurface Imaging with Emerging Signal Processing Techniques Behnam Jafarpour, USC

SPE Web Events
EXPLORATION • TECHNOLOGY • EDUCATION

#SPEWEBEVENTS

Large-Scale Connectivity Approximation

14 High-Resolution Subsurface Imaging with Emerging Signal Processing Techniques Behnam Jafarpour, USC

SPE Web Events
EXPLORATION • TECHNOLOGY • EDUCATION

#SPEWEBEVENTS

Transform-Domain Parameterization

Full-Rank Representation

$$\mathbf{u} = \sum_{i=1}^N \phi_i v_i$$

N expansion terms

Low-Rank Approximation

$$\hat{\mathbf{u}} \approx \sum_{s=1}^S \phi_s v_s$$

$S (<N)$ expansion terms

Questions to Address:

1) What class of basis functions ϕ_i ? (desirable properties)

2) Which (S) expansion terms to retain? **Novelty is in here!**

3) How to compute corresponding v_i 's?

4) History matching implementation?

15 High-Resolution Subsurface Imaging with Emerging Signal Processing Techniques Behnam Jafarpour, USC

SPE Web Events
EXPLORATION • TECHNOLOGY • EDUCATION

#SPEWEBEVENTS

Non-Geologic Compressive Basis Φ

- Discrete Cosine Transform (DCT)
 - Harmonics of cosine function (JPEG)
 - Jafarpour and McLaughlin, 2009, SPEJ
 - Bhark et al., 2010 JPSE
- Grid-Connectivity Transform
 - Eigen-vectors of grid-adjacency matrix
 - Bhark et al., 2010 WRR
 - Bhark et al., 2011 SPEJ
 - Generalized DCT for irregular geometry & unstructured grid
- Discrete Wavelet Transform (DWT)
 - Wavelets & filter banks (JPEG2000)
 - Sahni and Horne, 2005, SPERE
 - Jafarpour, 2010, IEEE-TGRS

16 High-Resolution Subsurface Imaging with Emerging Signal Processing Techniques Behnam Jafarpour, USC

#SPEWEBEVENTS

Geologic Compressive Basis Φ

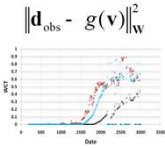
- Principle Component Analysis (PCA)
 - Leading eigenvectors of the given property covariance matrix
 - Gavalas, 1976, SPEJ
- Kernel-Based PCA (K-PCA)
 - PCA in nonlinear feature space
 - Sarma et al., 2008, MATHGEO
- Sparse Geologic Dictionaries
 - Learned sparse priors (K-SVD, MOD)
 - Aharon, 2006 IEEE-TSP
 - Khaninezhad et al., 2012 AWR

17 High-Resolution Subsurface Imaging with Emerging Signal Processing Techniques Behnam Jafarpour, USC

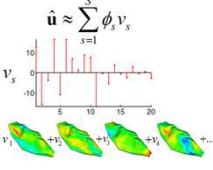
#SPEWEBEVENTS

Inversion with Reduced Parameters

(1) Flow Response



(2) Reduced Parameters



S-dimensional Minimization over $\mathbf{v}$

$$\hat{\mathbf{v}} = \arg \min_{\mathbf{v} \in \mathbb{R}^S} \left\{ \|\mathbf{d}_{\text{obs}} - \mathbf{g}(\mathbf{v})\|_w^2 + \dots \right\}$$

18 High-Resolution Subsurface Imaging with Emerging Signal Processing Techniques Behnam Jafarpour, USC

#SPEWEBEVENTS



Main Issues of Conventional Methods

- 1) Significant components are **pre-determined** (i.e., PCA)
- 2) Solutions **too smooth**
- 3) Sensitive to **uncertainty in prior continuity** model
- 4) Parameter reduction and **prior diversity** tradeoff
- 5) **Complex geology?** (meandering channels, etc.)

22

High-Resolution Subsurface Imaging with Emerging Signal Processing Techniques

Behnam Jafarpour, USC

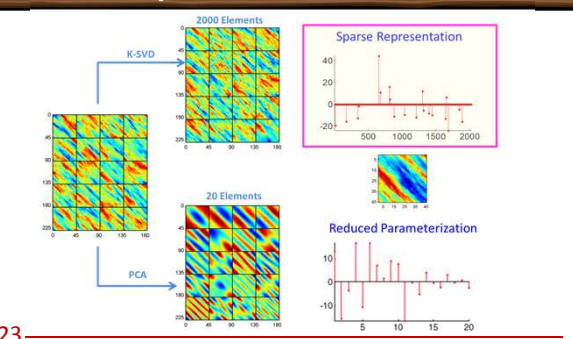


SPE Web Events

EXPLORATION • TECHNOLOGY • EDUCATION

#SPEWEBEVENTS

Sparse Reconstruction



23

High-Resolution Subsurface Imaging with Emerging Signal Processing Techniques

Behnam Jafarpour, USC

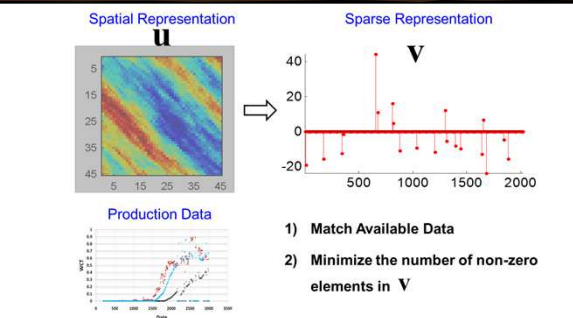


SPE Web Events

EXPLORATION • TECHNOLOGY • EDUCATION

#SPEWEBEVENTS

Solution Approach



24

High-Resolution Subsurface Imaging with Emerging Signal Processing Techniques

Behnam Jafarpour, USC



SPE Web Events

EXPLORATION • TECHNOLOGY • EDUCATION

#SPEWEBEVENTS

Solution Approach (Minimum Support)

(1) Sparsity of $\mathbf{v}$

$$\min_{\mathbf{v} \in \mathbb{R}^N} \|\mathbf{v}\|_0$$

Minimize solution support

(2) Incomplete Measurements

$$\mathbf{u}_M = \Phi_{M \times N} \mathbf{v}_N$$

(P₀) $\min_{\mathbf{v} \in \mathbb{R}^N} \|\mathbf{v}\|_0 \quad \text{s.t.} \quad \mathbf{u}_M = \Phi_{M \times N} \mathbf{v}_N$

L_0 -norm is discontinuous and non-convex (indicator function)
 Discrete (combinatorial) Optimization $\rightarrow$ NP-Hard
Heuristic approximate solutions have to be sought

25 High-Resolution Subsurface Imaging with Emerging Signal Processing Techniques Behnam Jafarpour, USC

Compressed Sensing Paradigm

Donoho 2006 (and others)

Compressed Sensing Theory

Minimum l_1 -norm reconstruction Donoho, Candes

$$\min_{\mathbf{v} \in \mathbb{R}^N} \|\mathbf{v}\|_1 \quad \text{subject to} \quad \mathbf{u}_M = \Phi_{M \times N} \mathbf{v}_N$$

Efficient LP solution (effectively identifies sparsity)

Perfect reconstruction with very high probability if:

- i) $\mathbf{v}$ is sufficiently sparse
- ii) Φ is nearly orthonormal
- iii) Sufficient measurements $\rightarrow M > C S \log(N/S)$

$$(1 - \delta_s) \|\mathbf{u}\|_2^2 \leq \|\Phi_s \mathbf{u}\|_2^2 \leq (1 + \delta_s) \|\mathbf{u}\|_2^2$$

Restricted Isometry Property (RIP)

26 High-Resolution Subsurface Imaging with Emerging Signal Processing Techniques Behnam Jafarpour, USC

Constructing Sparse Geologic Dictionaries

Aharon et al., 2006

Example: K-SVD Algorithm

Given a training data with L models $\mathbf{U}_{N \times L} = [\mathbf{u}_1 \dots \mathbf{u}_i \dots \mathbf{u}_L]$

Find a dictionary $\Phi_{N \times K}$ of size K that provides S -sparse representation $\mathbf{V}_{K \times L} = [\mathbf{v}_1 \dots \mathbf{v}_i \dots \mathbf{v}_L]$ of $\mathbf{U}$.

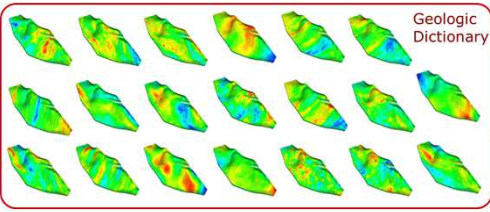
K-SVD Algorithm:
 Greedy algorithm that solves the following optimization

$$\{\hat{\Phi}, \hat{\mathbf{V}}\} = \arg \min_{\Phi, \mathbf{V}} \|\mathbf{U} - \Phi \mathbf{V}\|_F^2 \quad \text{s.t.} \quad \|\mathbf{v}_i\|_0 \leq S$$

$i = 1, 2, \dots, L$

27 High-Resolution Subsurface Imaging with Emerging Signal Processing Techniques Behnam Jafarpour, USC

Sparse Geologic Dictionary

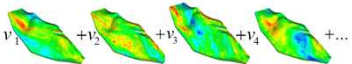


Geologic Dictionary

True Log-Perm



Only a small subset needed



$v_1 + v_2 + v_3 + v_4 + \dots$

28 High-Resolution Subsurface Imaging with Emerging Signal Processing Techniques Behnam Jafarpour, USC

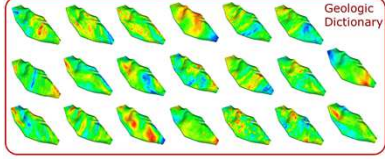


SPE Web Events

EXPLORATION • TECHNOLOGY • EDUCATION

#SPEWEBEVENTS

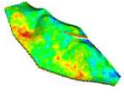
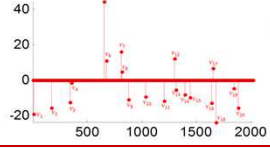
Sparse Approximation



Geologic Dictionary

$$\hat{u} = \sum_{i=1}^S \phi_i v_i$$

$S \ll N$ terms



29 High-Resolution Subsurface Imaging with Emerging Signal Processing Techniques Behnam Jafarpour, USC



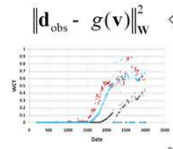
SPE Web Events

EXPLORATION • TECHNOLOGY • EDUCATION

#SPEWEBEVENTS

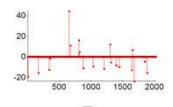
Sparsity-Promoting Inversion

(1) Production Data



$\|d_{obs} - g(v)\|_w^2$

(2) Sparsity of v



$\|v\|_0$

minimize

$$\hat{v} = \arg \min_{v \in \mathcal{H}^N} \left\{ \|d_{obs} - g(v)\|_w^2 + \lambda^2 \|v\|_p^p \right\}$$

$0 \leq p \leq 1$

Li and Jafarpour, 2010, Inverse Problem
Khaninezhad et al., 2011, Advances in Water Resources

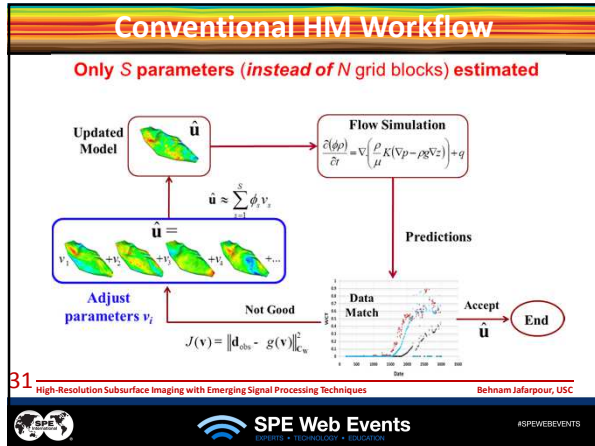
30 High-Resolution Subsurface Imaging with Emerging Signal Processing Techniques Behnam Jafarpour, USC

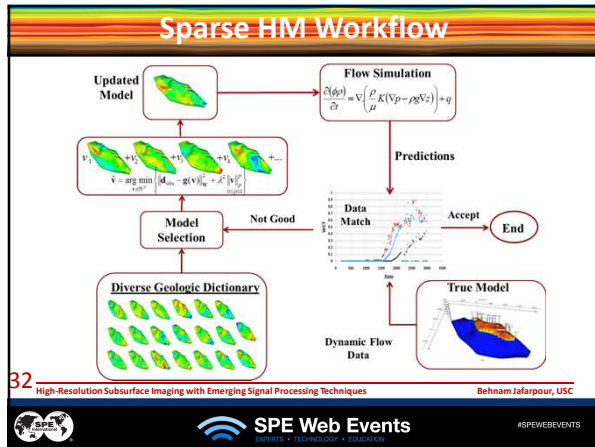


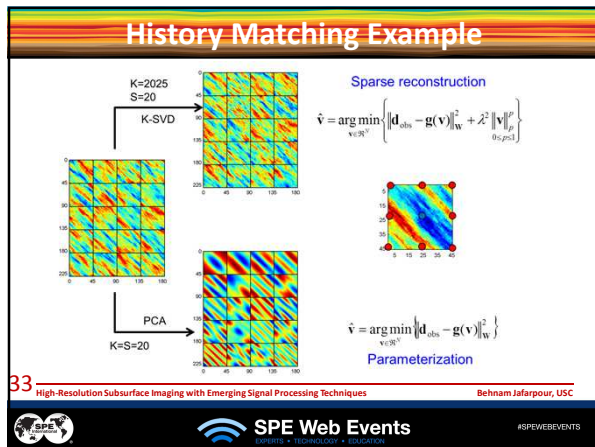
SPE Web Events

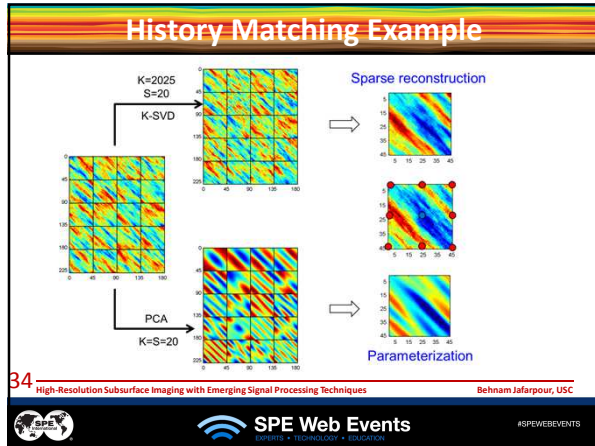
EXPLORATION • TECHNOLOGY • EDUCATION

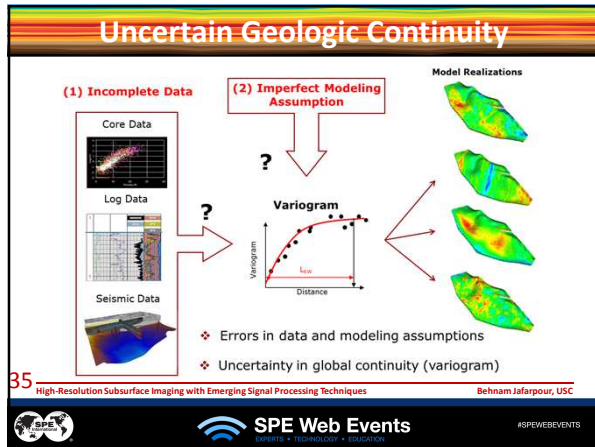
#SPEWEBEVENTS

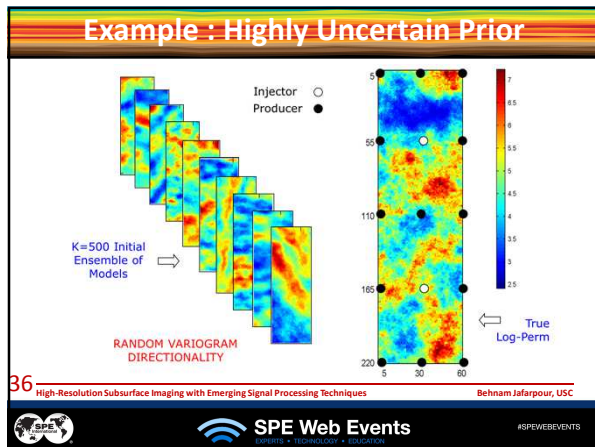


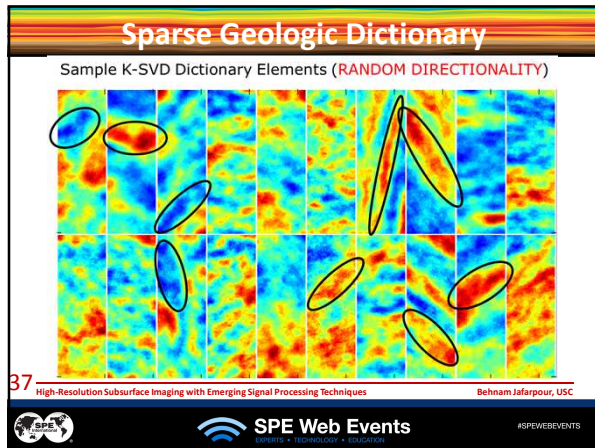


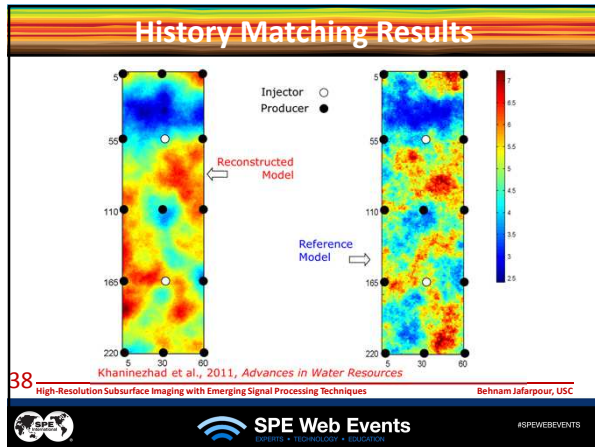












Conclusions

- ❑ Reservoir properties are **amenable to sparse representations**
- ❑ Sparse representations are useful for **model calibration** with sparsity-promoting methods
- ❑ **Compressed sensing** paradigm provides useful guidelines for model calibration under sparse priors
- ❑ **Sparse geologic dictionaries** are robust for model calibration under geologic uncertainty
- ❑ **Sparsity promoting inversion** has the potential to improve ill-posed model calibration inverse problems

39 High-Resolution Subsurface Imaging with Emerging Signal Processing Techniques Behnam Jafarpour, USC

SPE Web Events

Ongoing and Planned Work

- ❑ Prior model (error) identification for history matching under geologic uncertainty
- ❑ Uncertainty quantification using Bayesian formulations
- ❑ Efficient algorithms for constructing geologic dictionaries
- ❑ Innovative applications of sparse HM: adaptive sparsity, joint data/model sparsity, ...

40

High-Resolution Subsurface Imaging with Emerging Signal Processing Techniques

Behnam Jafarpour, USC



SPE Web Events

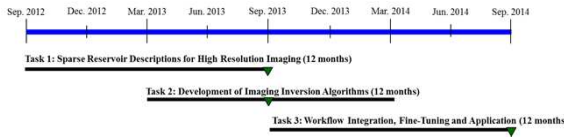
EXPLORATION • TECHNOLOGY • EDUCATION

#SPEWEBEVENTS

Project Schedule

Graduate Student:
M. Reza M. Khaninezhad (Ph.D., EE)

Tentative Schedule:



41

High-Resolution Subsurface Imaging with Emerging Signal Processing Techniques

Behnam Jafarpour, USC



SPE Web Events

EXPLORATION • TECHNOLOGY • EDUCATION

#SPEWEBEVENTS

Questions?

42

High-Resolution Subsurface Imaging with Emerging Signal Processing Techniques

Behnam Jafarpour, USC



SPE Web Events

EXPLORATION • TECHNOLOGY • EDUCATION

#SPEWEBEVENTS

Outline

- Brief review of fluid characterization
 - Gibbs free energy and EOS parameters
 - Volume shift parameters
- Perturbation from n-alkanes (PnA)
- Case studies
- Conclusions

Conventional Characterization Steps

1. Splitting C_{7+}

2. Estimating properties

Mole Fraction	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10		C100
Molecular weight	-	-	-	-	-	-	-	-	-	-	-	-
P _c	-	-	-	-	-	-	-	-	-	-	-	-
T _b	-	-	-	-	-	-	-	-	-	-	-	-
Density	-	-	-	-	-	-	-	-	-	-	-	-
Volume shift parameter	-	-	-	-	-	-	-	-	-	-	-	-

3. Grouping components and determining properties

Mole Fraction	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11	P12	P13	P14	P15
Molecular weight	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
P _c	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
T _b	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Density	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Volume shift parameter	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

4. Tuning the EOS to data

Ungrouped
Grouped components

SPE Web Events
Society • Technology • Innovation

#SPEWEBEVENTS

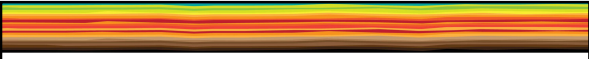


Common Questions/Issues

- What should I do with so many adjustment parameters?
 - Implicit relationship between phase behavior predictions and adjustment parameters
- I matched some PVT data, but am not sure if this EOS model is reliable in gasflooding simulation.
 - Characterization under unavoidable uncertainties
- Why are different methods used for different fluids?



#SPEWEBEVENTS



Objective

- Universal characterization method that works for heavy oils, volatile oils, gas condensates, and near-critical fluids.
 - Peng-Robinson EOS with van der Waals mixing rules
 - Multiphase behavior; oleic, gaseous, solvent-rich liquid, and aqueous phases
- The focus of the first year was on the normal oleic-gaseous phase behavior.



#SPEWEBEVENTS



Compositional Reservoir Simulation

- Mass conservation equations for components
 - Local equilibrium assumption
- Phase behavior solution
 - Gibbs free energy is minimized at T, P, and overall composition for each gridblock

Fluid characterization is to build a Gibbs free energy using an EOS.
Comp. res. simulator then minimizes the Gibbs free energy at T, P, and z on the fly.



#SPEWEBEVENTS

Gibbs Free Energy, Peng-Robinson EOS

$$\underline{G}_R = \sum_{i=1}^{N_c} z_i \ln(z_i \bar{\phi}_i)$$

z_i : Overall concentration of component i
 $\bar{\phi}_i$: Fugacity coefficient of component i in a mixture
 N_c : Number of components

$$P = \frac{RT}{\underline{V}-b} - \frac{a(T)}{\underline{V}(\underline{V}+b)+b(\underline{V}-b)}$$

$a = \Omega_a \frac{(RT_c)^2}{P_c} \alpha(T)$: Attraction parameter
 $b = \Omega_b \frac{RT_c}{P_c}$: Covolume parameter

van der Waals mixing rules are used for mixtures.

#SPEWEBEVENTS

Overview of Phase Behavior Modeling

Steps	Input	Output
1. a and b for components	$(T_c, P_c, \omega)_i$	a_i, b_i
2. a and b for mixtures	Binary interaction parameters van der Waals mixing rules	a_{mix}, b_{mix}
3. Gibbs free energy	Peng-Robinson EOS	Gibbs free energy
4. Compositional behavior (Flash calculation)	Temperature, Pressure, Composition	Phase compositions and amounts
5. Volumetric behavior	Volume-shift parameters	Volumetric phase properties

Match?
PVT data

Gibbs free energy depends on volume shift through T_c , P_c , ω , and BIPs.

#SPEWEBEVENTS

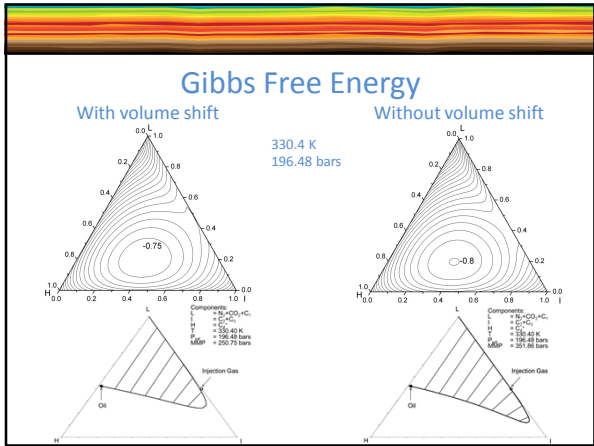
Effect of Volume Shift on Gibbs Energy

- An oil is characterized using three components with/without volume shift
- Two characterizations give the same predictions of
 - Saturation pressure 196.48 bars at 330.4 K
 - Oil densities at pressures between 7.1 bars and 276.3 bars at 330.4 K
- However, they have different properties for the H component

Comp.	Mole Fractions	MW (gm/mol)	PR EOS with volume shift				PR EOS without volume shift		
			T_c (K)	P_c (bars)	ω	C_{PEN} (cc/mol)	T_c (K)	P_c (bars)	ω
L	0.4219	16.10	190.86	46.07	0.0089	-5.19	190.86	46.07	0.0089
I	0.0176	31.66	315.59	47.83	0.1065	-5.85	315.59	47.83	0.1065
H	0.5605	291.30	848.33	24.18	0.4373	-110.14	1022.44	19.48	0.5108

Temperature (K) 330.40
Saturation Pressure (bars) 196.48

L= N₂+CO₂+C₁, I= C₂+C₃, H= C₄+



Perturbation from n-Alkane (PnA)

- Paraffin-naphthene-aromatic (P-N-A) distribution of a plus fraction is interpreted as a perturbation from 100% n-alkanes
- No volume shift
- No change in EOS and mixing rules

CCCCCCCC

n-Nonane

C1CCC(CC1)C

1,2-Dimethylcyclohexane

CC1CCC(CC1)C

2,5-Dimethylheptane

C1=CC=CC=C1

Ethylbenzene

Pedersen and Christensen 2007

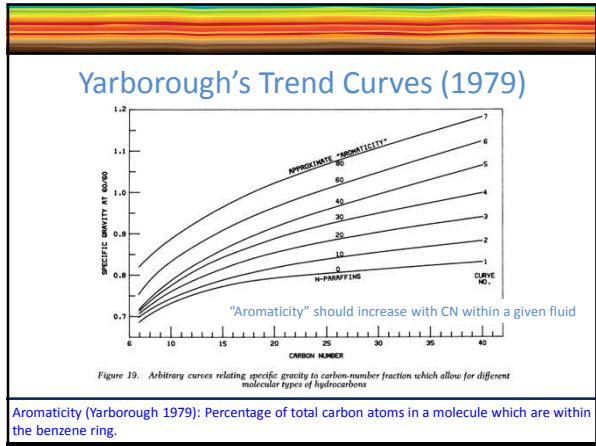
New Correlations for n-Alkanes for PR EOS

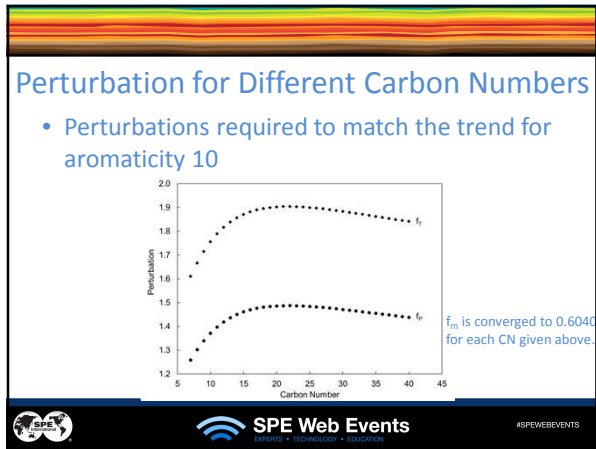
Kumar and Okuno (2012) Fluid Phase Equilibria

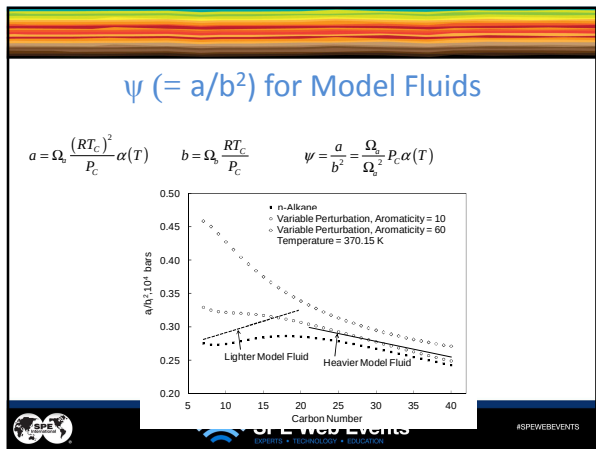
$$T_c = 1154.35 - 844.83(1.0 + 1.7557 \times 10^{-3} MW)^{-2.0}$$
$$P_c = 559.93(MW)^{-0.638} - 1.49$$
$$m = 0.4707 + 2.4831(MW)^{\frac{29.932}{MW}}$$
$$m = m(\omega) \text{ as defined by Peng and Robinson (1978)}$$

$$MW \leftarrow f_T MW$$
$$MW \leftarrow MW / f_P$$
$$MW \leftarrow f_m MW$$

- They give 3.0% and 3.4% AADs in density and vapor pressure predictions, respectively, for n-alkanes from C₇ to C₁₀₀.
- Conventional correlations give AADs of 87% for density prediction and 61% for vapor pressure prediction for n-C₁₀₀.
- T_c and P_c of n-alkanes are lower than those of other hydrocarbons for a given carbon number. The trend is the other way around for ω.







Characterization Steps

- Composition characterization
 - E.g., chi-squared distribution fitted to composition data
- Perturbation from n-alkanes
 - f_T , f_p , and f_m
 - One more parameter γ to control the trend of ψ

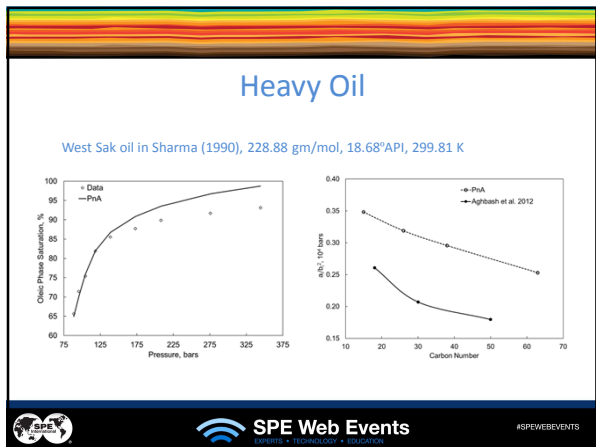
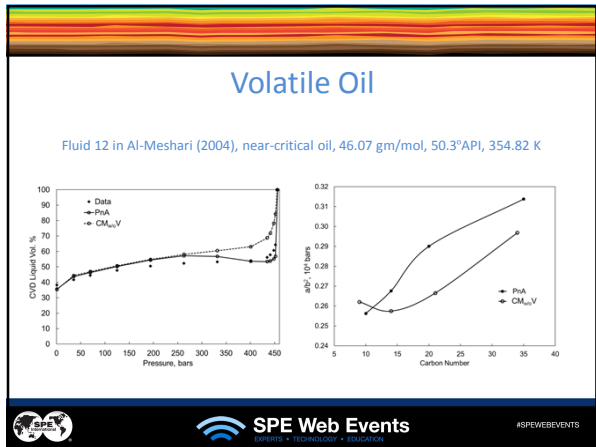
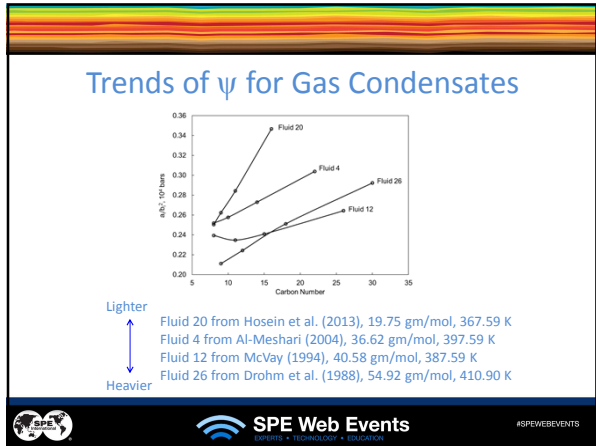
Case Studies

- 77 different reservoir fluids
 - Heavy oils, volatile oils, gas condensates, and near-critical fluids
- Saturation pressure and densities at T_{res}
- Other data are used to test the predictions
- Four pseudo components
- Four adjustment parameters
 - f_T , f_p , f_m , and γ

Gas Condensates

Liquid volume calculation for constant-volume depletion
Fluid 10 in Al-Meshari (2004), Near-critical gas condensate, 39.23 gm/mol

β : Mole fraction of vapor phase



Conclusions

- Volume-shift parameters affect compositional behavior predictions when used as regression parameters.
- The PnA method
 - f_p , f_D , f_m , and γ
 - No volume shift
 - PR EOS with van der Waals mixing rules

#SPEWEBEVENTS

Conclusions

- Perturbation should consider the CN range.

#SPEWEBEVENTS

Papers to be presented at ATCE 2013

- SPE 166135 “Universal Fluid Characterization using an EOS based on Perturbation from n-Alkanes”
- SPE 166345 “Mass Transfer on Multiphase Transitions in Low-Temperature CO₂ Floods”
- SPE 166371 “A New Method for Modeling Bypassed Oil Recovery in EOS Compositional Simulation”

#SPEWEBEVENTS

Effect of Volume Shift

- Effect on Gibbs free energy

$$g^m = \left(\frac{\partial \Delta G}{\partial T} \right)^m = \sum_{i=1}^N x_i \ln \left[\frac{h_i^m(T, P, \{x\})}{h_i^m(T_0, P_0, \{x\})} \exp \left(\frac{P - P_0}{RT} \right) \right]$$
$$g^m = \left(\frac{\partial \Delta G}{\partial T} \right)^m = \sum_{i=1}^N x_i \ln \left[\frac{h_i^m(T, P, \{x\})}{h_i^m(T_0, P_0, \{x\})} \exp \left(\frac{P - P_0}{RT} \right) \right] = \sum_{i=1}^N x_i \ln \left[\frac{h_i^m(T, P, \{x\})}{h_i^m(T_0, P_0, \{x\})} \exp \left(\frac{P - P_0}{RT} \right) \right]$$

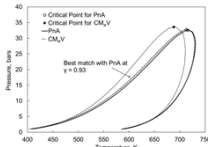
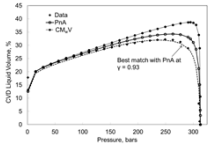
- Effect on fugacity equations

$$f_i^m(T, P, \{x\}) = y_i^m(T, P, \{y\}) \phi_i^m(T, P, \{y\})$$
$$f_i^m(T, P, \{x\}) = y_i^m(T, P, \{y\}) \phi_i^m(T, P, \{y\})$$

#SPEWEBEVENTS

Gas Condensates

Liquid volume calculation for constant-volume depletion
Fluid 11 in Al-Meshari (2004), Near-critical gas condensate, 41.71 gm/mol, 424.82 K



#SPEWEBEVENTS

Questions?

#SPEWEBEVENTS

Full-Resolution Simulations of Multiscale Processes

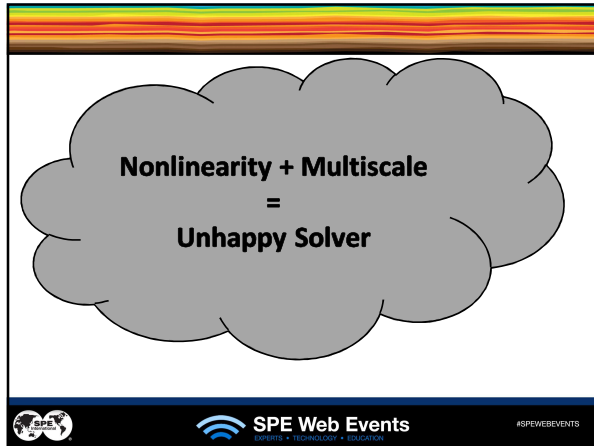
Rami M Younis
Assistant Professor
McDougall School of Petroleum Engineering
The University of Tulsa

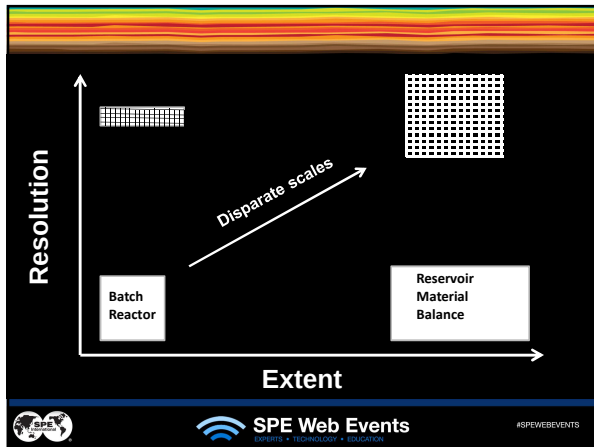
#SPEWEBEVENTS

#SPEWEBEVENTS

Nonlinearity and Complexity

#SPEWEBEVENTS









ADAPTIVITY and LOCALITY

#SPEWEBEVENTS



Two types of “adaptive strategy”


$$\frac{\partial a(p)}{\partial t} + F(p, \nabla p, \Delta p) = 0$$

↓

Type A: AMR, AIM, Fractional Stepping, Operator splitting

$$R(p^{n+k}, p^{n+k-1}, \dots, p^n; \delta_t, \delta_x) = 0$$

↓

Type B: Nonlinear and linear preconditioners, localized solvers

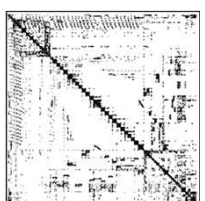
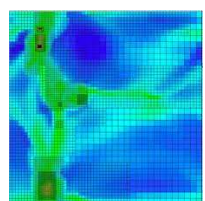
p^n

#SPEWEBEVENTS



Examples

TYPE A: Adaptive Meshing **TYPE B:** Localized solvers



#SPEWEBEVENTS

Both strategies need to know:

Define $\delta(x, t) = |p(x, t) - p(x, t + \epsilon)|, \epsilon > 0$.

Need $\Omega = \text{supp } \delta(x, t)$

Or if, $\tilde{\delta}(x, t) \geq \delta(x, t) \geq 0$,

Can use $\tilde{\Omega} = \text{supp } \tilde{\delta}(x, t) \subseteq \Omega$

Recall, $\text{supp } f(x) = \{x: f(x) \neq 0\}$



SPE Web Events

#SPEWEBEVENTS

The precise spatiotemporal support of change in reservoirs

- The specific problem of interest
- Mathematical derivation
- Examples
- The future for multiscale models



SPE Web Events

#SPEWEBEVENTS

Canonical form of problem of interest:

Coupled flow and transport

$$\frac{\partial}{\partial t} a(p, Z) + \nabla[\mathbf{K}(p, Z) \nabla p] + w(p, Z) = 0$$

Flow state: $p(x, t)$

Transport states: $Z(x, t)$



SPE Web Events

#SPEWEBEVENTS

1

FLOW

Pressure distribution

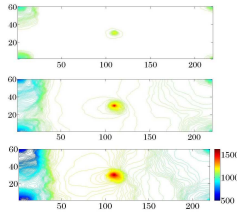
Infinite domain-of-dependence

Travelling (Very) Fast

Parabolic Character

Potential

$$\frac{\partial}{\partial t} a(p) + \nabla[K(p) \cdot \nabla \psi(p)] + w(p) = 0$$



 **SPE Web Events**
EXPERIENCE • TECHNOLOGY • EDUCATION

#SPEWEBEVENTS

2

TRANSPORT

Travelling Waves

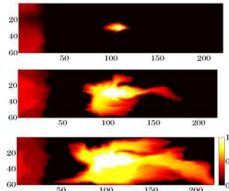
Finite domain-of-dependence

Hyperbolic Character

Entropy Solutions

Rankine-Hugoniot

$$\frac{\partial a(Z)}{\partial t} + \nabla f(Z) = q(Z)$$



 **SPE Web Events**
EXPERIENCE • TECHNOLOGY • EDUCATION

#SPEWEBEVENTS

How to get $\tilde{\Omega} = \text{supp } \tilde{\delta}(x, t)$!

 **SPE Web Events**
EXPERIENCE • TECHNOLOGY • EDUCATION

#SPEWEBEVENTS

Using the fully-implicit semidiscrete form

Seek $p^n(x)$ and $Z^n(x)$ for $t^n = n\Delta t, n = 0, 1, \dots$

$$\begin{aligned} R(p^{n+1}, Z^{n+1}) \\ = a(p^{n+1}, Z^{n+1}) + \Delta t \nabla [K(p^{n+1}, Z^{n+1}) \nabla p^{n+1}] \\ + w(p^{n+1}, Z^{n+1}) \end{aligned}$$



SPE Web Events

#SPEWEBEVENTS

Solve nonlinear PDE using Newton's Method!

Solve,

$$R(p^{n+1}(x)) = 0$$

First guess,

$$[p^{n+1}]^{\mu=0} = p^n$$

Then,

$$R'([p^{n+1}]^\mu) \delta^\mu = -R([p^{n+1}]^\mu)$$

and,

$$\delta^\mu = [p^{n+1}]^{\mu+1} - [p^{n+1}]^\mu$$



SPE Web Events

#SPEWEBEVENTS

There we have it...

Since,

$$\delta^\mu = [p^{n+1}]^{\mu+1} - [p^{n+1}]^\mu,$$

and,

$$[p^{n+1}]^{\mu=0} = p^n,$$

We have,

$$\sum_{\mu=0}^K \delta^\mu = \delta(x, t^n)$$



SPE Web Events

#SPEWEBEVENTS

Flow with frozen transport

Solve

$$R(p^{n+1}) = a(p^{n+1}) + \Delta t \nabla [K(p^{n+1}) \nabla p^{n+1}] + w(p^{n+1})$$

For each μ solve $R'([p^{n+1}]^\mu) \delta^\mu = -R([p^{n+1}]^\mu)$

A conservative estimate $\tilde{\delta} \geq \delta^\mu$:

$$\Delta \tilde{\delta} - \lambda^2 \tilde{\delta} = R(p^\mu)$$

where

$$\lambda = \inf \sqrt{\frac{a'(p^\mu)}{\Delta t K(p^\mu)}}$$



#SPEWEBEVENTS

The screened Poisson equation,

$$\Delta \tilde{\delta} - \lambda^2 \tilde{\delta} = R(p^\mu)$$

has the Fundamental Solution,

$$\Phi(x) = \begin{cases} -\frac{1}{2\lambda} \exp(-\lambda|x|) & x \in \mathbb{R}, \\ -\frac{1}{2\pi} K_0(\lambda|x|) & x \in \mathbb{R}^2, \\ -\frac{1}{4\pi} \frac{\exp(-\lambda|x|)}{|x|} & x \in \mathbb{R}^3 \end{cases}$$

So given a corrector function, h , the solution is

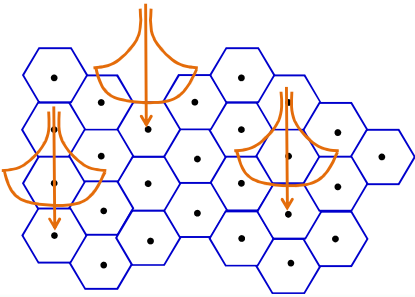
$$\tilde{\delta} = \int_D [\Phi(x-y) - h(y)] R \, dy$$

$$\leq \sup R \int_D [\Phi(x-y) - h(y)] \, dy$$



#SPEWEBEVENTS

Algorithm:





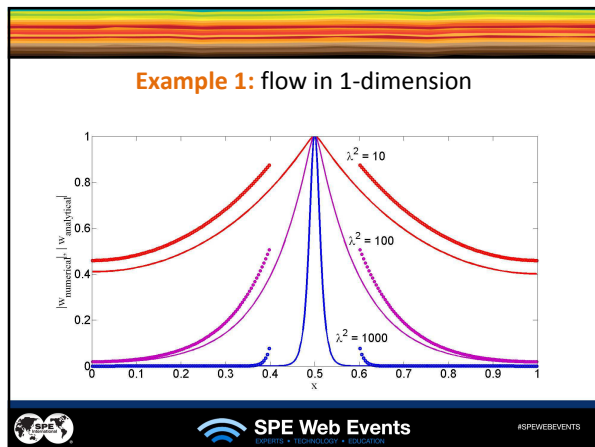
#SPEWEBEVENTS

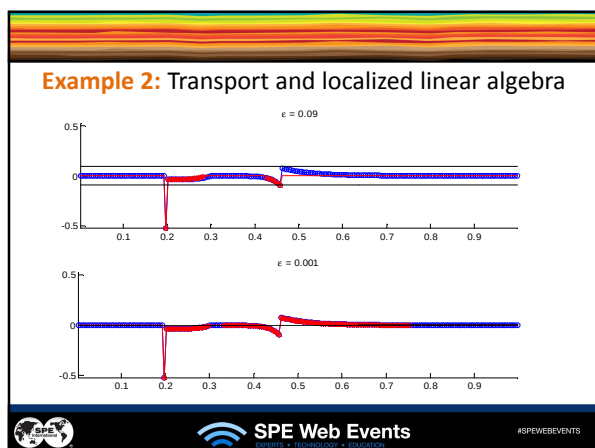


Computational Examples



#SPEWEBEVENTS





A future for full-resolution multiscale

- Type A methods:
 - Exploit locality in finite volume multiscale methods
 - AMR that knows exactly where and when to adapt
- Type B methods
 - Full-resolution Cartesian grid with solver that knows about locality!

SPE Web Events
EXPERTS • TECHNOLOGY • EDUCATION

#SPEWEBEVENTS

Thank You

Rami M Younis
ryounis@utulsa.edu

SPE Web Events
EXPERTS • TECHNOLOGY • EDUCATION

#SPEWEBEVENTS
